OPTIMAL CONSUMPTION AND DEFORESTATION

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This paper presents farmer's optimization problem and how it relates to his or her deforestation behavior. The main technique used here is a stochastic control model solved by the Hamilton-Jacobi-Bellman (HJB) equation. The farmer's capital accumulation is treated as a jump process approximated by the Poisson process. The control variables are assumed to be of Markovian type. The ARMA(1,1) deforestation model of this paper has given further insight into farmers' deforestation and capital accumulation behavior. Of particular importance is how farmers' attitude towards risks (and their precautionary motives of saving) affect these behaviors. A risk-taking farmer appears to be more likely to clear a forest than a risk-averting one. The result also shows the case where consumption optimalization may preclude further forest clearing. But given the limitations of the modeling, these results should be seen as only a part of a much more complex deforestation behavior.

INTRODUCTION

In the previous study[1], the deforestation model was built without reference to the farmer's utility function. Consumption was set ex ante at a given subsistence level and no optimization was undertaken. While such an approach has produced a model capable of explaining the empirical results (Wibowo, D.H., Tisdell, C.A. and Byron, R.N., 1997), it would be desirable to see if these empirical results can also be explained by a model built on the basis of the farmer's consumption optimization. For this reason, this paper will focus on the farmer's optimization problem and how it relates to his or her deforestation behavior. The main technique used here is the stochastic control model which will be solved by the Hamilton-Jacobi-Bellman (HJB) equation. The farmer's capital accumulation is treated as a jump process approximated by the Poisson process. The control variables are assumed to be of Markovian type.

The main strategy adopted here is to first determine the optimal level of the control variable u_i. As in Wibowo (2009), these controls represents the proportion of income allocated for consumption $C_t = u_{1t}X_t$, income-generating expenses $Q_t = u_{2t}X_t$, and forest clearing expenses $L_t = u_{3t}X_t$. The income residual (or saving) is given by $\tilde{X}_t = u_{4t}X_t$, where $u_{4t} = 1 - \sum_i u_{it}$. The next step is to show that if the jump process does not occur in a given time interval, than the optimized L_t will fall below *B*, which is the minimum cash capital required to clear a forest and establish a *ladang.* This is the first necessary condition for deforestation in the study area. However, as discussed in Wibowo, D.H. (2009), the farmer would not be financially capable of clearing a forest if his or her income falls below a "target boundary" $\bar{X} = B + \bar{C} + \bar{Q}$. Here $\bar{C} \leq C_t$ denotes consumption at the subsistence level, while $\bar{Q} \leq Q_t$ represents the minimum level of income-generating expenses. Consequently we have $L_t = 0$ if $L_t < B$ and/or $X_t < \bar{X}$.

Before proceeding further with the modeling, a brief discussion on the technique is presented. Readers who wish to go beyond the basic materials described here are recommended to consult *inter alia* Malliaris and Brock (1982) and Oksendal (1992)

[1] See Wibowo, D.H. and Byron, R. N. (1999); Wibowo, D.H. (1999); Wibowo, D.H., Tisdell, C.A. and Byron, R.N. (1997); and Wibowo, D.H. (2009),

THE HAMILTON-JACOBI-BELLMAN EQUATION

Consider an SDE

$$d X_t = A(t, X_t)\, dt + \sigma(t, X_t)\, d B_t, \tag{1}$$

Where $X_t \in \mathcal{R}^n$, $A(t, X_t) \in \mathcal{R}^n$, $\sigma(t, X_t) \in \mathcal{R}^{n \times m}$, and B_t is *m*-dimensional Brownian motion. For this SDE, we can define the differential generator (or weak infinitesimal operator) $\mathcal{H}$ of X_t as follows :

$$Hf(t,x) = \lim_{\Delta t \downarrow 0} \frac{E^{t,x}(f(t+\Delta t, X_{t+\Delta t})) - f(t,x)}{\Delta t}; x \in R^n \tag{2}$$

Where $E^{t,x}$ denotes expectation w. r. t. $P^{t,x}$, that is, the probability law of B_t starting at (t, x).

Now let $\mathcal{D}_{\mathcal{H}}(t,x)$ be the set of functions $f : \mathcal{R}^n \to \mathcal{R}$.such that the limit exists at (t, x), and $\mathcal{D}_{\mathcal{H}}$ be the set of functions for which the limit exists for all $t, x \subset R^n$. We can then state the following theorem.

Theorem 1. *If $f \in G^2$ is bounded with first and second derivatives, then for the SDE (1), $f \in \mathcal{D}_{\mathcal{H}}$ and*

$$\mathcal{H} f(t,x) = \frac{\partial f}{\partial t} + \sum_i A_{i(t,x)} \frac{\partial f}{\partial x_1} + \frac{1}{2} \sum_{i,j} (\sigma \sigma^T)i,j\,(t,x) \frac{\partial^2 f}{\partial x_i \partial x_j}. \tag{3}$$

Proof : See Øksendal (1992), pp. 93-95 for the proof.

As an example of the use of Theorem 1, consider the Itô diffusion version of the Langevin equation: $dC_t = (-u_1 C_t + u_2)dt + vdB_t$

Example 2. *For the SDE*

$$dC_t = -u_1 C^t\, dt + vdB_t, \tag{4}$$

The differential generator $\mathcal{H} f(c)$ is then

$$\mathcal{H} f(c) = -u_1 C f_c + \frac{1}{2} v^2 f_{cc}, \tag{5}$$

Where f_c and f_{cc} are the first and second partial derivatives of f w.r.t. c.

Another example is given below.

Example 3. *The Itô diffusion whose differential generator $\mathcal{H} f(x)$ is given by*

$$\mathcal{H}f(x) = 2x_2 f x_1 + In\,(1 + x_1^2 + x_2^2) f_{x2} + \frac{1}{2} x_1^2 f_{x_1^2} + x_1 f_{x1\,x2} + \frac{1}{2} f_{x_2^2},$$

Satisfies

$$\begin{pmatrix} dX_1 \\ dX_2 \end{pmatrix} = \begin{pmatrix} 2x_2 \\ 1n\,(1 + x\,\frac{2}{1} + x\,\frac{2}{2}) \end{pmatrix} dt + \begin{pmatrix} x_1 \\ 1 \end{pmatrix} dB_t.$$

Now suppose that the system in equation (1) is controlled by a parameter $u \in U \subset R^d$ whose value is chosen from a given Borel set U at any instant.

Equation (8.1) then becomes

$$dX_t = A\left(t, X_t, u\right)dt + \sigma\left(t, X_t, u\right)dB_t. \tag{6}$$

Note here that $u = u\,(t,\omega)$ it self is a stochastic process, and the function $\omega \to u\,(t,\omega)$ is $\mathcal{F}_t$ -adapted.

Furthermore, assume that we need to maximize the following objective function :

$$\max E^{s,x}\left[\int_s^T \emptyset\,(t, X_t, u)dt + J(T, X_T)\right] \tag{7}$$

Where as before $E^{s,x}$ is the expectation w. r. t. the probability law of B_t starting at (s, x), given that $0 \leq s \leq t,\ X_s = x$. As usual, the utility function $\emptyset\,(t,\ X_t, u)$ and the bequest function $J(T,\ X_T)$ are both assumed to be strictly concave in their respective X arguments. Here T denotes the exit time of X_T

From equation (7), one has the performance criterion of the form

$$J\,(s, x, u) = E^{s,x}\left[\int_s^T \emptyset\,(\tau, X_\tau, u)d\tau + J(T, X_T)\right] \tag{8}$$

Where $\emptyset$.is the utility function.

Our problem is then to find a value for $J^*(s, x, u)$ and $u^*(s,\omega)$ such that

$$J^*(s,x,u) = \sup_{u \in U} j(s,x,u) = j(s,x,u^*) \tag{9}$$

Assuming that the value of u at time h depends only on the state of the system at this time, not on the initial values $s = 0$ and $X_0 = x$, then we have u as a Markov control. For $v \in \mathcal{U}$, we use Theorem 1 to define

$$(\mathcal{H}^v f)(y) = \frac{\partial f}{\partial s}(y) + \sum_i A_i\,(y, v)\,\frac{\partial f}{\partial x_i} + \frac{1}{2}\sum_{i,j}(\sigma\sigma^T)i,j\,(y, v)\,\frac{\partial^2 f}{\partial x_i \partial x_j} \tag{10}$$

Where $y = (s, x)$.

Applying the theory of stochastic control, we have the HJB equation as follows :

Theorem 4. (The Hamilton-Jacobi-Bellman Equation) *Define $J^*(y,u)$ as in equation (8.9), where u is a Markou control. Also define*

$$\Upsilon = \sup_{v \in U}[F(y,v) + (H^v J^*)(y)] \quad (11)$$

If J^* is twice continuously differentiable and an optimal Markou control u exists, then u satisfies

$$\Upsilon = 0 \quad (12)$$

for all $y \neq (T, X_T)$, and

$$J^*(T, X_T) = J(T, X_T) \quad (13)$$

for $y = (T, X_T)$.

Proof : See Øksendal (1992), pp. 182-184 for the proof.

Here note that $\mathcal{F}(y,v)$ denotes the objective function to be optimized, which in the case of equation (7) is given by $\emptyset(t,x,u)$.

Now suppose that we add a jump process to equation (1). If the jump is modeled as a Poisson process, the SDE (1) becomes

$$dX_t = A(t,X_t)dt + \sigma(t,X_t)dB_t + S(t,X_t)dq_t, \quad (14)$$

Where the other notations are as in equation (1), $S(t,X_t) \subset R^n$ is a function and $q(t)$ a Poisson process. For simplicity, we assume that there is no correlation between the Brownian motion and the Poisson process. Let

- $\lambda\Delta t + O(\Delta t)$ be the probability that q_t jumps once in the time interval $(t, t+\Delta t)$,
- $1 - \lambda\Delta t + O(\Delta t)$ be the probability that q_t does not jump in the time interval $(t, t+\Delta t)$
- $O(\Delta t)$ be the probability that q_t jumps more than once in the time interval $(t, t+\Delta t)$, and
- the amplitude S of the jump be a random variable with density function $p(\bar{q})$,

where λ is the mean number of the jump occurrences per unit time, Δt can be as small as we like, and $O(\Delta t)$ is the asymptotic order symbol defined by $\psi(\Delta t)$ is $O(\Delta t)$ if

$$\lim_{\Delta t \downarrow 0} \left(\frac{\psi(\Delta t)}{\Delta t}\right) = 0$$

Then, for the SDE (14) we have the differential generator

$$\mathcal{H}f(t,x) = \frac{\partial f}{\partial t} + \sum_i A_i\,(t,x)\frac{\partial f}{\partial x_i} + \frac{1}{2}\sum_{i,j}(\sigma\sigma^T)\,i,j(t,x)\frac{\partial^2 f}{\partial x_i \partial x_j} + \lambda\left(\int_{\bar{q}}\left(f(t,x+S(t,x)\,\bar{q}) - f(t,x)\right)p(\bar{q})d\bar{q}\right) \tag{15}$$

After assuming that the $O(\Delta t)$ vanishes. Substituting the generator (15) into equation (11) one can use Theorem 4 to solve a stochastic control problem involving a mixed Brownian motion and Poisson process.

Suppose that we have a set of optimal Markovian controls v_i where $\sum_i v_i = 1$ Taking the view of $\sum_i v_i = 1$ as a constraint, equation (11) can be expressed as a standard Lagrangian equation

$$\mathcal{L} = \Upsilon + \bar{\lambda}\,(1 - \textstyle\sum_i u_i) \tag{16}$$

Where $\bar{\lambda}$ is the multiplier. The optimal controls $v_i \in \mathcal{u}$ can then be found from the first-order conditions

$$\frac{\partial \mathcal{L}}{\partial u_i} = \frac{\partial \mathcal{L}}{\partial \bar{\lambda}} = 0. \tag{17}$$

Needless to say that this Lagrangian technique is applicable for equation (11) regardless of whether the differential generator takes the form of (3) or (15).

STOCHASTIC CONTROL MODELS FOR DEFORESTATION

The Multiplicative Verhulst Model

Initially an attempt was made to determine optimal controls for a system characterized by the multiplicative Verhulst model. Using the technique of separation of variable, however, there appear to be no simple closed form solutions for this model. Thus, another specification of the system will be used in the later part of this paper.

To see why stochastic control problems that use the multiplicative Verhulst model are difficult to solve, let us reconsider the SDE

$$d\,X_t = (aX_t - bX_t^2)dt + (gX_t - hX_t^2)dB_t, \tag{18}$$

Where, as in equation (33), $a = u_2 \in (0,1)$, $b = u_2\theta/K$, $g = \alpha\theta$, $h = \alpha\theta^2/K$ and $\theta = 1 - (1+r)(1 - \sum_{i=1}^{3} u_i)$. For brevity, the SDE (18) is assumed to contain no jumps in the process X_t.

Next define the performance criterion

$$J\,(t,x,u) = E^{t,x}\left[\int_t^T e^{-\rho s}\,\emptyset\,(c,l,s)ds\right] \tag{19}$$

With a bequest function

$$J(T,x,u) = 0, \tag{20}$$

Where ρ denotes the rate of time preference, c .is consumption, and l is the size of land cleared from a forest. The relationship between l_t and L_t is given by $l_t = L_t/B = u_{3t}X_t/B$, where B is the unit cost of forest clearing after being normalized against the price of consumption. Note that here the price of consumption is normalized into unity.

Applying the differential generator (3), equation (10) and Theorem 4, we have

$$e^{-\rho t}\emptyset + J_t\,(ax - bx^2)J_x + \frac{1}{2}\,(gx - hx^2)^2 J_{xx} = 0 \tag{21}$$

Given the constraint $\sum_{i=1}^{4} v_i - 1$, one obtains the following first-order conditions from the Lagrangian (8.16) by differentiating w. r. t. v_i :

$$\bar{\lambda} = e^{-\rho t}\phi_c x - \frac{(1+r)}{K}\,v_2 x^2 J_x + \tilde{\lambda}$$

$$\bar{\lambda} = xJ_x - \frac{1}{K}\left[1 + 2(1+r)v_2 - (1+r)(1 - v_1 - v_3)\right]x^2 J_x + \tilde{\lambda}$$

$$\bar{\lambda} = e^{-\rho t}\phi_l \frac{x}{B} - \frac{(1+r)}{K} v_2 x^2 J_x + \tilde{\lambda}$$

$$\bar{\lambda} = \frac{(1+r)}{K} v_2 x^2 J_x - \tilde{\lambda}, \qquad (22)$$

And

$$1 - \sum_{i=1}^{4} v_i = 0. \qquad (23)$$

Here the subscripts c, l and x denote partial derivatives w. r. t. these variables, and

$$\tilde{\lambda} = \alpha^2\theta(1+r)x^2(1-\frac{\theta}{K}x)(1-\frac{2\theta}{K}x)J_{xx}. \qquad (24)$$

From the firs and the third rows of equation (22) we have

$$\phi_c = \frac{\phi_l}{B}, \qquad (25)$$

While from the first and the second ones we have upon simplification

$$\frac{\theta}{K} = \frac{J_x - e^{-\rho t}\phi_c}{xJ_x}, \qquad (26)$$

Where $\theta = 1 - (1+r)(1 - \sum_{i=1}^{3} v_i)$.

Meanwhile, an expression for v_2 is given by

$$v_2 = \frac{e^{-\rho t}\phi_c K}{2(1+r)\, \mathcal{X} J_x} - \alpha^2 K^2 \left(\frac{e^{-\rho t}\phi_c}{J_x} - 1\right)^2 \left(\frac{e^{-\rho t}\phi_c}{J_x}\right) \frac{J_{xx}}{\mathcal{X} J_x}, \qquad (27)$$

after substituting and rearranging equation (26) and the first and the fourth row of equation (22).

From the second and the fourth rows of equation (22) one obtains

$$v_4 = \frac{e^{-\rho t}\phi_c K}{2(1+r)\, \mathcal{X} J_x} + \frac{K}{(1+2)\mathcal{X}} - \frac{1}{(1+r)J_x}. \qquad (28)$$

Now define $\beta = v_1 + v_3$ Because $\sum_{i=1}^{4} = 1$ then we have from (27) and (28) that

$$\beta = 1 + \alpha^2 K^2 \left(\frac{e^{\ \rho t}\phi_c}{J_x} - 1\right)^2 \left(\frac{e^{\ \rho t}\phi_c}{J_x}\right) \frac{J_{xx}}{\mathcal{X} J_x} + \frac{K}{(1+2)\mathcal{X}} - \frac{1}{(1+r)J_x}. \qquad (29)$$

Substitute (26) and (27) into equation (21) to have the following partial differential equation (PDE)

$$\phi + J_t + \frac{e^{-\rho t}\phi_c^2 K}{2\,(1+r)J_x} + \alpha K\left(\frac{e^{-\rho t}\phi_c}{J_x} - 1\right)^2 \left(\frac{e^{-\rho t}\phi_c^2}{J_x^2}\right)\left(\frac{1}{2} - \alpha K x\right) J_{xx} = 0. \qquad (30)$$

To see the difficulties in solving (30), let

$$\emptyset(c,l,t) = f(t)\overline{\emptyset}(c,l)$$
$$= f(t)\overline{\emptyset}(v_1 x, \frac{v_3}{B} x), \qquad (31)$$

Where $v_{1,}v_3$ and B assumed to be constants. Taking the derivatives of (31) w. r. t. c and x, and applying equation (25) gives

$$\emptyset_c = \frac{\emptyset_x}{v_1 + v_3} = \frac{f\,\overline{\emptyset}_x}{v_1 + v_3}. \qquad (32)$$

Substituting (32) into (29) gives …. As the solution of

$$\beta = 1 + \left[\alpha^2 K^2 \left(K - \frac{1}{2}\right)\left(e^{-\rho t} f\overline{\emptyset}_x - \beta J_x\right)^2 \left(e^{-\rho t} f\overline{\emptyset}_x\right)\beta^{-3} X J_x^{-4} J_{xx}\right] - \frac{K}{(1+r)x} + \frac{1}{(1+r)J_x}. \qquad (33)$$

We can see from equations (32) and (33) that $\emptyset_{c,}\emptyset_x$ .and β .are linked by a non-linear relationship. Thus, there appears to be no easy closed form solution to the PDE (30). Consequently, we may need to resort to numerical methods to solve the problem. Because numerical methods are not the focus of this thesis, and because in any case the problem is a higly non-linear parabolic problem which is difficult to solve, the methods are not adopted here. Instead, another form of SDEs is examined in order to find a closed form solution for the problem. Given its popularity in the economics profession, the ARMA(1,1) specification is then adopted in the next subsection.

The ARMA(1,1) Model

From equation $c_t = a_0 + a_1 c_{t-1} + e_t - (1-\varphi)e_{t-1}$ and $dc_t = (-u_1 c_t + u_2)d_t + (1-\alpha)v dB_t$, we know that the ARMA (1.1) specification can be transformed into a generalization of the Langevin equation. Assuming that the noise appears multiplicatively and there is a jump process in the system, then we have

$$dX_{2t} = v_2(-b_1 X_t + b_2)dt + v_2(1-\alpha)\sigma X_t dB_t + Ke^{-\gamma t} X_t dq_t \qquad (34)$$

as a modal for the farmer's stochastic income process. Here b_i denotes drift coefficients related to the AR parameters, α is the moving average coefficient, while the control variable v_i are as defined in the second paragraph of this paper. To represent the farmer's capital accumulation, a jump process $Ke^{-\gamma t} X_t dq_t$ is added, where K is a constant representing the amplitude of the jump, γ is decaying factor and q_t is a Poisson process. As usual, the Brownian motion and the Poisson process are assumed to have no correlation, where $\overline{q} \equiv 1.w.p.1$. The readers can easily see that we have an (upward) jump that decays over time. Finally, note that in order to keep things simple, no correlation between the jump process and the control variable is assumed. In this case, one may argue that such an assumption seems unnecessary because we can, say, put a v_2 term in the jump term of equation (34). But the matter is not that simple. Empirically, the jump is not reliant on the control variables in the current time. However, if the jump occurs, farmers might reorganize their income allocation as they now have a large amount of cash. Thus, the jump might in fact affect the control, not the other way around. Such an empirical fact cannot be represented by simply putting a v_2 term in the jump term of equation (34). For this reason, the above assumption is adopted in order to have a greater tractability.

Following the same procedure used to construct equation (33), we have

$$dX_t = v_2(-b_1\theta X_t + b_2)dt + v_2(1-\alpha)\sigma X_t dB_t + Ke^{-\gamma t} X_t dq_t \qquad (35)$$

Where, as before, $\theta = 1-(1+r)(1-\bar{\theta})$.

Now define the performance criterion and the bequest function as in equations (19) and (20), respectively. Appling the differential generator (15) and Theorem 4, we have

$$\Upsilon := e^{-\rho t}\emptyset + J_t + v_2(b_2 - b_1\theta\mathcal{X})J_x + \frac{1}{2}v_2^2(1-\alpha)^2\sigma^2 x^2 J_{xx} + \lambda\,[J(t, ke^{-\gamma t}x) - J(t, x)] = 0 \qquad (36)$$

The first-order conditions of this problem are

$$\begin{aligned} \bar{\lambda} &= e^{-\rho t}\emptyset_c\mathcal{X} - b_1 v_2(1+r)\mathcal{X}J_x \\ \bar{\lambda} &= b_2 - b_1(1-(1+r)v_4)\mathcal{X}J_x + v_2(1-\alpha)^2\sigma^2 x^2 J_{xx} \\ \bar{\lambda} &= e^{-\rho t}\emptyset_l\frac{\mathcal{X}}{B} - b_1 v_2(1+r)\mathcal{X}J_x \\ \bar{\lambda} &= (1+r)b_1 v_2\mathcal{X}J_x, \end{aligned} \qquad (37)$$

And

$$1 - \sum_{i=1}^{4} v_i = 0, \qquad (38)$$

Where the subscripts c, l .and x denote partial derivatives w. r. t. these variables.

From the first and the third rows of equation (37) we have

$$\emptyset_c = \frac{\emptyset_l}{B} \qquad (39)$$

While from the first and the fourth ones we have

$$v_2 = \frac{e^{-\rho t}\emptyset_x}{2(1+\mathrm{r})b_1 J_x} \tag{40}$$

Assuming that $\emptyset$ takes the form of equation (31), applying (39) gives

$$\emptyset_c = \frac{\emptyset_x}{v_1 + v_3} = \frac{f\,\overline{\emptyset}_x}{v_1 + v_3} \tag{41}$$

From equations (40) and (41) we have

$$(1+r)(v_1 + v_3) = \frac{e^{-\rho t}\emptyset_x}{2\mathrm{v}_2\, b_1 J_x} \tag{42}$$

And

$$\theta = 1 - (1+r) + \frac{e^{-\rho t}\emptyset_x}{2\mathrm{v}_2\, b_1 J_x} + (1+r)v_2. \tag{43}$$

From the first and the second rows of the first-order conditions (37) we have

$$v_4 = \frac{1}{(1+r)J_x}\left[\frac{e^{-\rho t}\emptyset_x}{v_1 + v_3} - v_2\,\left(b_1(1+r)J_x + (1-\alpha)^2\sigma^2 \mathcal{X} J_{xx}\right) - \frac{b_2}{x} + b_1 J_x\right]. \tag{44}$$

Substitute equations (40),(41), and (43) into (36) and rearrange to have the PDE

$$\Upsilon := \emptyset + \frac{J_t}{e^{-\rho t}} + \frac{\emptyset_x b_2}{2(1+\mathrm{r})\mathrm{b}_1(v_1+v_3)} - \frac{x\emptyset_x}{2(1+\mathrm{r})(v_1+v_3)} + \frac{x\emptyset_x}{2(v_1+v_3)} - \frac{e^{-\rho t}x\phi_x^2}{4(1+r)v_2 b_1(v_1+v_3)J_x} - \frac{x\emptyset_x v_2}{2(v_1+v_3)}$$
$$+\frac{\mathrm{v}_2^2(1-\alpha)^2\sigma^2\mathrm{x}^2 J_{xx}}{2e^{-\rho t}} + \lambda\,[J(t, ke^{-\gamma t}x) - J(t,x)] = 0 \tag{45}$$

To parameterize the PDE (45), we use the isoelastic utility function: $v(c_t) = \frac{c_t^{1-\delta}}{1-\delta'}$, where land is treated as a durable good that enter the farmer's utility function. Thus, let

$$\emptyset(t, c, l) = f(t)\left(\frac{c^{1-\delta}}{1-\delta} + \frac{l^{1-\delta}}{1-\delta}\right)$$
$$= f(t)\left[v_1^{1-\delta} + \left(\frac{v_3}{B}\right)^{1-\delta}\right]\frac{\mathcal{X}^{1-\delta}}{1-\delta'} \tag{46}$$

Where, as shown in equation (13), δ is the coefficient of relative risk aversion Also let

$$J = f(t)e^{-\mu t}v_1{}^{\delta}\frac{x^{1-\delta}}{1-\delta} \tag{47}$$

After taking the partial derivatives of $\emptyset$ w. r. t. its c, l .and x arguments, from equation (39) we have

$$v_1 + v_3 = v_1\left(1 + B^{\bar{\delta}}\right), \tag{48}$$

Which can also be written as

$$v_1^{1-\delta} + \left(\frac{v_3}{B}\right)^{1-\delta} = v_1^{1-\delta}\left(1 + B^{\bar{\delta}}\right), \tag{49}$$

Where $\bar{\delta} = \frac{\delta - 1}{\delta}$

Use equations (40, 41, 46, 47, 48) and (49) to derive

$$v_2 = \frac{1}{2(1+r)b_1}, \tag{50}$$

Which is the solution for optimum v_2

Now use equations (46, 47, 48, 49) and (50) to solve the PDE (45). After rearranging and eliminating the same terms we have

$$f_t = D_0 f + D_1, \tag{51}$$

Where

$$D_0 = \rho - v_1\left(1 + B^{\bar{\delta}}\right) - (1-\delta)\left(\frac{r}{2(1+r)}\right) + \frac{(1-\alpha)^2\sigma^2\delta + (1-\delta)}{8(1+r)^2 b_1^2} + \frac{1}{2}v_1^{\bar{\delta}+1}\left(1 + B^{\bar{\delta}}\right)(1-\delta) + \lambda e^{-\rho t}\left(\bar{k}e^{-\bar{\gamma}t} - 1\right) \tag{52}$$

And

$$D_1 = \frac{1-\delta}{4(1+r)b_1} \tag{53}$$

Note that $\bar{k} = k^{1-\delta}$ and $\bar{\gamma} = \gamma(1-\delta)$. If $\lambda = 0$ and/or $\bar{\gamma} = \ln \bar{k}/t$, then the jump disappears.

To solve (51) we use the general form

$$f(t) = M_o e^{D_0 t} + M_1 t,$$

Where M_O and M_1 are constants. Using the boundary condition $J(T, x) = 0$ we arrive at the following solution for $f(t)$

$$f(t) = D_1\left[\frac{e^{-D_0(T-t)}}{D_0T-1} - \frac{1}{D_0t-1}\right] \tag{54}$$

For D_0T-1 and $D_0t-1 \neq 0$. Consequently. We have

$$\emptyset = (t,c,l) = D_1\left[\frac{e^{-D_0(T-t)}}{D_0T-1} - \frac{1}{D_0t-1}\right]\left[v_1^{1-\delta} + \left(\frac{v_3}{B}\right)^{1-\delta}\right]\frac{x^{1-\delta}}{1-\delta}, \tag{55}$$

and

$$J(t,x,v) = D_1\left[\frac{e^{-D_0(T-t)}}{D_0T-1} - \frac{1}{D_0t-1}\right]e^{-\rho t}\, v_1^{-\delta}\frac{x^{1-\delta}}{1-\delta}. \tag{56}$$

The solution of v_2 has been given by equation (50). For the other control variables, the solutions are

$$v_4 = \frac{(1-\alpha)^2\sigma^2\delta + b_1 - \frac{1}{2}}{1+r} \tag{57}$$

After substituting relevant expressions into equation (44), and

$$v_1 = \frac{\left[1 - \frac{1}{2(1+r)b_1} - \frac{(1-\alpha)^2\sigma^2\delta + b_1 - \frac{1}{2}}{1+r}\right]}{1 + B^{\frac{1}{\delta}}} \tag{58}$$

Due to (48 and 49) and the fact that $v_1 + v_3 = 1 - v_2 - v_4$. With v_1, v_2, and v_4 solved, the solution for optimum $v_3 = 1 - v_1 - v_2 - v_4$ can be easily found. Here it should be noted that the expression for optimum v_1 is also influenced by the cost of forest clearing B. Note that since we require $v_{1=1,2} \in (0,1)$.and $v_{1=3,4} \in [0,1)$, this requirement will impose certain constraints on the problem parameters.

Equation (58) is, however, only relevant for the cases where $L_t \geq B$ after the optimal value of v_3 is substituted into the system, and where $X_t \geq \overline{X}$. If the opposite is true, that is, in the case of $L_t < B$ and/ or $X_t < \bar{X}$, then the farmer in question is not financially capable of clearing a forest. Consequently, the optimum v_3 is set to zero, and v_1 becomes

$$v_1 = 1 - \frac{1}{2(1+r)b_1} - \frac{(1-\alpha)^2\sigma^2\delta + b_1 - \frac{1}{2}}{1+r} \quad (59)$$

Finally, it is worth noting that the optimal values of the controls are independent of the jump process. This result comes as a consequence of our assumption of no correlation between the controls and the jump process.

Numerical Simulation

As in Wibowo (2009), consider the case of an *anak ladang* in the upper Kerinci region. Using the empirical data reported in Wibowo (2009), we have $\bar{X}$ = 2.51, *B* =1.69, and $u_1 = 0.40$ and $u_2 =$ 0.57. If u_1 and/or u_2 are assumed to represent the optimal values of the control variables, we have $v_1 =$ 0.40 and/or $v_2 =$ 0.57. Substituting this value of v_2 into equation (48) gives an "empirical value" of $b_1 =$ 0.84, if $r =$ 0.05 is assumed.

To represent the case of a risk-taking (or risk-loving) farmer, we assume $\delta = -3$. The case of risk-averting farmer is given by $\delta = 0.01$. The value of $\delta = 3$ is not chosen because it leads to the sum of the controls exceeding unity.

Finally, the value of X is normalized into unity. The value of X with a jump is set at $X =$ 3.26 because the lump sum received by the farmers by the end of the cinnamon planting cycle is 2.26 larger than the regular potato income. Note here that $\lambda = 1$ has been implicitly assumed[2].

The results of the simulation are presented in Table 1. Case 1 shows the "standard" case for a risk-taking farmer, where the optima v_1 and v_2 are assumed to be equal to the empirical values of u_1 and u_2. Because $L <$ B and $X < X$, we have the case where the farmer cannot afford the cost of forest clearing and its associated cost. As expected, with a relatively large negative value of δ, the control v_1 would fall sharply had the farmer cleared a forest.

[2] The jump can also be approximated by choosing a value of k and γ and then computing J $(t, ke^{-\gamma t}x) - J(t,x)$ by use of equation (56). This approach is not adopted here for time-efficiency reason

Table 1. Numerical Simulations of the Control Variables

		Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
A	The Parameters						
	b1	0.84	0.84	0.84	0.84	0.84	0.84
	alpha	1.32	1.10	1.32	1.10	1.32	1.10
	sigma	1.00	1.00	1.00	1.00	1.00	1.00
	delta	-3.00	0.01	-3.00	0.01	-3.00	0.01
	r	0.05	0.05	0.05	0.05	0.05	0.05
	B	1.69	1.69	1.69	1.69	1.69	1.69
	X	1.00	1.00	3.26	3.26	7.00	7.00
	Xbar	2.51	2.51	2.51	2.51	2.51	2.51
B	The Controls						
	V1-def	0.13	0.11	0.13	0.11	0.13	0.11
	v1-nodef	0.40	0.11	0.40	0.11	0.40	0.11
	v2	0.57	0.57	0.57	0.57	0.57	0.57
	v3	0.27	0.00	0.27	0.00	0.27	0.00
	v4	0.03	0.32	0.03	0.32	0.03	0.32
C	The Results						
	C-def	0.13	0.11	0.44	0.36	0.94	0.77
	C-nodef	0.40	0.11	1.31	0.36	2.82	0.77
	Q	0.57	0.57	1.86	1.86	3.99	3.99
	L	0.27	0.00	0.88	0.00	1.88	0.00
	Saving	0.03	0.32	0.09	1.04	0.19	2.24

Notes:
b 1 = drift coefficient,
alpha, sigma = diffusion coefficients,
delta = the coefficient of risk aversion,
r = rate of interest
B = the cost of forest clearing and dry-land farming establishment,
X = income, X =1 represents the case of no jumps, X=Y represents the case where the jump is (Y-1) times the original income,
X bar = the minimum amount of cash capital required to clear a forest and to cover other associated costs,
V 1-def = the optimal proportion of income allocated for consumption if the farmer clears a forest,
V1-nodef = the optimal proportion of income allocated for consumption if the farmer does not clear a forest,
V2 = the optimal proportion of income allocated for income-generating activities,
V3 = the optimal proportion of income allocated for forest clearing, v3=0 if farmers does not clear a forest,
V4 = the optimal proportion of income allocated for saving,
C = consumption,
Q = income-generating expenses
L = forest clearing expenses, and
Saving = the amount of income saved.

Case 2 exhibits an interesting result as far as forest conservation is concerned. For this risk-averting farmer, an optimal strategy would be to set $v_3 = 0$, and consequently, $L = 0$. As can seen from Table 1, this strategy also includes reducing his or her current consumption. More importantly, even if this farmer receives a much larger income (see Cases 4 and 6), he or she still sets $v_3 = 0$. This result from the fact that $v_3 = v_{1\,/B^{\bar{\delta}}}$, where in this case $\bar{\delta} = 99$. With such a large exponent for B, the $v_3 = 0$ result follows.

Another important result can be seen from Case 3. With a jump of 2.26 times the regular income, the optimal L is shown to be only 0.88, much less than the minimum requirement of $B = 1.69$. This is despite the fact that $X > \bar{X}$. Only if jump increases to around six times the regular income does the optimal L exceed B . This result can be seen from Case 5. Note that in all Cases 1,3 and 5, v_3 is always greater than v_1 if the farmer clears a forest, but is always lower than v_1 in the no-deforestation case.

DISCUSSION

The model indicates that, because land is treated as an argument in the farmer's utility function, than the farmer also considers the cost of forest clearing in their optimal consumption and forest clearing decisions. If the farmer's optimal allocation for forest clearing exceeds the minimum costs required *and* his total income *(X)* exceeds the target income $\bar{X}$, then he or she has the financial capacity to clear a forest.

The results of out numerical simulations, however, shows that capital accumulation (as represented by the jump process) does not always result in the farmer having the financial capacity to clear a forest. In the case of a risk- averting farmer, for example, the farmer tends to put more money into his or her saving, while at the same time setting his or her allocation for forest clearing to zero. One explanation for this is that for a risk-averting farmer, forest clearing is seen as too-risky investment. Thus, he or she prefers to have a "liquid" saving (such as cash and jewelry) which can be liquidated quickly. Because this farmer is also prepared to sacrifice his or her consumption in return for increased saving, the farmer represents the classical case of precautionary saving. In other words, precautionary motives may work against further deforestation.

Another explanation for this result is that the model used here might have been too restrictive, in the sense that it does not allow the control variables to vary according to the jump process. But as discussed earlier, such an assumption is needed for both tractability and empirical reasons . In addition to this, the (time-additive) CRRA utility function used might have also contributed to this restrictiveness because the function itself is restrictive despite its popularity. Furthermore, the value of δ chosen, i.e. $\delta = 0.01$ seems also to have caused this result. Unfortunately, such a low positive value is needed in order to satisfy the constraint $v_i = 1,2 \in (0, 1)$ and $v_i = 3,4 \in [0, 1)$. Because of all these problems, the result should be viewed only as a special case that might not be applicable under other circumstances.

The case of a risk-taking farmer gives further insight into farmers' deforestation behavior. This type of farmer is prepared to sacrifice consumption in order to invest in forest clearing. This can be seen from the fact that if this farmer clears a forest, his or her optimal consumption declines sharply to a level lower than the amount of money allocated for forest clearing. Thus, as far as forest conservation is concerned, risk-taking farmers are more likely to pose a threat to forest reserves than risk-averting ones. The "cat-and-mouse" game discussed in Wibowo, D.H., Tisdell, C.A. and Byron, R.N. (1997), provides empirical supports for this result.

The simulation, however, also shows that if risk-taking farmers optimize their income allocation, capital accumulation does not necessarily give them financial capacity to clear a forest. This is despite the fact that their total income exceeds a "target boundary" representing the minimum level of income required to cover forest clearing costs, consumption at the subsistence level and the minimum level of income-generating expenses. If this result is compared to that of the Fokker-Planck modeling, it is clear that consumption optimalization has raised the financial requirement for forest clearing. In the Fokker-Planck modeling, if total income exceeds the target boundary, then the forest clearing costs are already covered. This is because the consumption is set at the subsistence level. With optimalization, the consumption moves above the subsistence level, resulting in less money allocated for forest clearing costs. As a result, the farmer's optimal level of forest clearing expenses may fall below the costs of forest clearing.

While this result shows that there are cases where consumption optimalization precludes deforestation, there are other cases where some farmers are prepared to live at the subsistence level in order to own land later in life. This may on the surface gives an indication that deforestation is caused more by sub-optimal consumption than by the optimal one. However, such a finding may well result from our underestimation of the importance of land in the farmer's utility function. The farmer may in fact give a much greater weight to land than previously assumed, in him or her being prepared to sacrifice more consumption. Thus, the farmer's choice to live at the subsistence level may in fact be an optimal one. For this reason and other shortcomings discussed earlier, the results of this paper should be seen only as a precursor to a more complex deforestation modeling.

CONCLUSION

The ARMA(1,1) deforestation model of this paper has given further insight into farmers' deforestation and capital accumulation behavior. Of particular importance here is how farmers' attitude towards risks (and their precautionary motives of saving) affect these behaviors. A risk-taking farmer appears to be more likely to clear a forest than a risk-averting one. The result also shows the case where consumption optimalization may preclude further forest clearing. But given the limitations of this modeling, these results should be seen as only a part of a much more complex deforestation behavior.

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